energy balance, and the definition of the bulk mean temperature, one obtains

$$\theta_s(X, Z) = -4X - Z^2 + \frac{Z^4}{4} + \frac{7}{24} + \frac{RW}{2q_w}\left[4X + Z^2 - \frac{Z^4}{2} - \frac{1}{4}\right] \quad (8)$$

from which it immediately follows that the asymptotic Nusselt number is

$$N_{Nu} = \frac{2}{\frac{11}{24} - \frac{RW}{8q_w}}$$

Asymptotic solutions for arbitrary n may be obtained in exactly the same way.

The solution to Equation (7) may now be obtained as usual by letting

$$\theta(X, Z) = \theta_s(X, Z) - \theta_1(X, Z) \quad (9)$$

where θ_1 satisfies the homogeneous form of Equation (6) and the boundary conditions which follow from Equations (7). Now let

$$\theta_1 = \sum_{i=0}^{\infty} B_i\, e^{-a_i X}\, \varphi_i\,(Z) \quad (10)$$

where $\varphi_i(Z)$ are solutions to Equation (3) with zero derivatives at the wall and the center of the tube. Since a_0 is zero for this problem, in view of Equation (8), it is easily shown that B_0 is zero. Hence i may be considered to take values from 1 to ∞ without changing Equation (10). These, a_i, and $\varphi_i(Z)$, have been calculated (6). The B_i are given by

$$B_i = \frac{\int_0^1 \varphi_i(Z)\,[\theta_s(O, Z) - \theta(O, Z)]\,Z(1 - Z^2)\,dZ}{\int_0^1 Z(1 - Z^2)\,\varphi_i^2\,(Z)\,dZ} \quad (11)$$

Using Equation (3) and the boundary conditions for $\varphi_i(Z)$ which are determined from Equations (7) one can easily show that

$$\int_0^1 Z(1 - Z^2)\,\varphi_i(Z)\,dZ = 0$$

$$i = 1, 2 \ldots . n \quad (12)$$

If $\theta(O, Z)$ is chosen to be the asymptotic solution obtained from an inlet section of arbitrary length with a constant heat flux q_{w1}, and if a step change in heat flux from q_{w1} to q_{w2} occurs at $X = 0$, then with Equation (12) it follows that

$$B_i = (1 - q_{w1}/q_{w2})\,B_i' \quad (13)$$

Obviously if the inlet section is well insulated, $q_{w1} = 0$ and

$$B_i = B_i'$$

Therefore the B_i' previously calculated (6) may be used directly for the more general problem involving heat generation. Again Equation (13) is general for arbitrary $f(Z)$ for constant heat flux for the same reasons stated in the previous section, and only a new θ_s need be determined as Equation (8) was.

NOTATION

- a = dimensionless constant, Toors Equation (33)
- a_i = eigenvalues
- B_i = dimensionless coefficient, Equations (4) and (11)
- d_w = tube diameter, (ft.)
- $f(Z)$ = dimensionless energy generation function
- h = local heat transfer coefficient, B.t.u./(hr.)(sq.ft./°F.)
- J = conversion factor = 778 ft.-lb.-force/B.t.u.
- k = thermal conductivity, B.t.u./(hr.)(ft./°F.)
- n = dimensionless constant in power law model
- N_{Nu} = Nusselt number hd_w/k
- p = pressure, (lb.-force/sq.ft.)
- q_w = wall heat flux, B.t.u./(hr.-sq. ft.)
- r = radial coordinate, (ft.)
- R = radius of tube, (ft.)
- T = temperature, (°R.)
- T_w = wall temperature (°R.)
- T_b = bulk mean temperature, (°R.)
- U = local velocity, (ft./hr.)
- U_m = mean velocity, (ft./hr.)
- W = mean net volumetric rate of heat generation across tube, B.t.u./(hr.)(cu.ft.)
- x = axial coordinate
- X = reduced axial coordinate $(n/n + 2)\ (\alpha x/R^2U_m)$
- Z = reduced radial coordinate (r/R)

Greek Letters

- α = thermal diffusivity, (sq.ft./hr.)
- θ = reduced temperature
- θ_s = asymptotic solution to Equations (1) and (6)
- θ_1 = homogeneous solution to Equations (1) and (6)
- φ_i = eigenfunctions

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PETROCHEMICALS AND PETROLEUM REFINING, Vol. 57, No. 34, 1961.

Liquid Hydrogen—the Ultimate Fuel, Peter C. Vander Arend and B. J. Ferro. This paper deals mainly with the general design and operating features of the tonnage liquid hydrogen facility designed, built, and operated by Air Products, Incorporated, for the United States Air Force at West Palm Beach, Florida. **Reforming Heavy Catalytically Cracked Gasoline for High Octane Blend Stock,** R. F. Kress, H. W. Nagel, and H. E. Reif. Current trends indicate that the leaded research octane number of premium motor fuels may increase over the next ten years to about the 105 level. The average refinery has a definite shortage of

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gasoline streams whose octane numbers are high enough for incorporation in such a premium fuel. **Development of the Model II Fluid Coker,** H. N. Weinberg, R. O. Wright, and A. L. Saxton. This paper briefly describes the history of petroleum coking leading up to fluid coking and covers in some detail the most recent advance in this field, the development of Model II Fluid Coking. **Recent Developments in the Technology of Residue Processing,** H. Beuther, J. B. McKinley, and R. A. Flinn. The nature and molecular composition of residues are discussed, and methods for residue processing by thermal, catalytic, and separation techniques are reviewed. **The TCC Airlift,** Jean M. Bourquet, Robert D. Drew, and Stephen Valentine, III. The theoretical and practical aspects of lift design and operation are included in this paper, which covers the dimensions, materials of construction, cost, and operating characteristics of a commercial lift. **The Modern Hydrogen Fluoride Alkylation Unit,** E. R. Fenske. Various changes in design which have made improvements in the octane quality of the product alkylate possible are discussed, such as the introduction of the isostripper, reductions in specific catalyst consumption, greatly minimized corrosion problems, and improvements in operating flexibility. **Dynamic Adsorption of Isobutane and Isopentane on Silica Gel,** G. H. Dale, D. M. Haskell, H. E. Keeling, and L. A. Warzel. Dynamic-adsorption studies on silica gel were made with isobutane- and isopentane-enriched natural gas at high flow rates. Chromatographic analyses reported for effluent composition from the pilot-scale equipment illustrate the effect of particle size, bed depth, gas velocity, and concentrations upon recovery. **Natural Gas Processing Developments,** John E. Walke and Charles E. Webber. Natural gas processing now encompasses a number of additional operations to remove water vapor, carbon dioxide, hydrogen sulfide, and helium, and this paper deals with these developments. **Appraisal of Potentials for Petrochemical Manufacture in the Southwest,** John M. Dale. The Southwest does not promise to become the major consuming center for petrochemical end-products in the United States, but it does have a large percentage of the raw materials needed by the petrochemical industry. While the cost of these raw materials will increase, the pattern of discovery and the off-shore area promise to make the supply of these raw materials more abundant in the future. **The Production of Alpha-Olefins,** Raymond A. Franz. Since this class of compounds is playing such an important role as an intermediate in the petrochemical field, an investigation of a thermal cracking process was initiated and a study of the nature of the olefins was made. This investigation is still in progress, and many questions are yet unanswered. **Hydrogenation Techniques at Combined High Temperatures and Pressures,** E. B. Shultz, Jr., H. L. Feldkirchner, and E. J. Pyrcioch. Novel high-pressure pilot plant and large bench-scale equipment have been developed for processing solid and liquid fossil fuels by destructive hydrogenation. Typical results obtained in coal-char and oil-shale hydrogasification tests are given to illustrate the range of application of the techniques. **Carbon Dioxide Removal by Hot Potassium Carbonate and Amine Scrubbing,** H. S. Trail, J. C. Reynolds, and R. E. Alexander. This paper describes a plant where the known difficulties of existing plants were carefully considered in the initial design. The design basis and performance are given for a carbon dioxide removal system employing hot potassium carbonate scrubbing as the bulk removal step and monoethanolamine scrubbing as the final cleanup.